Determining the correct quadrant of an angle

Complex number:

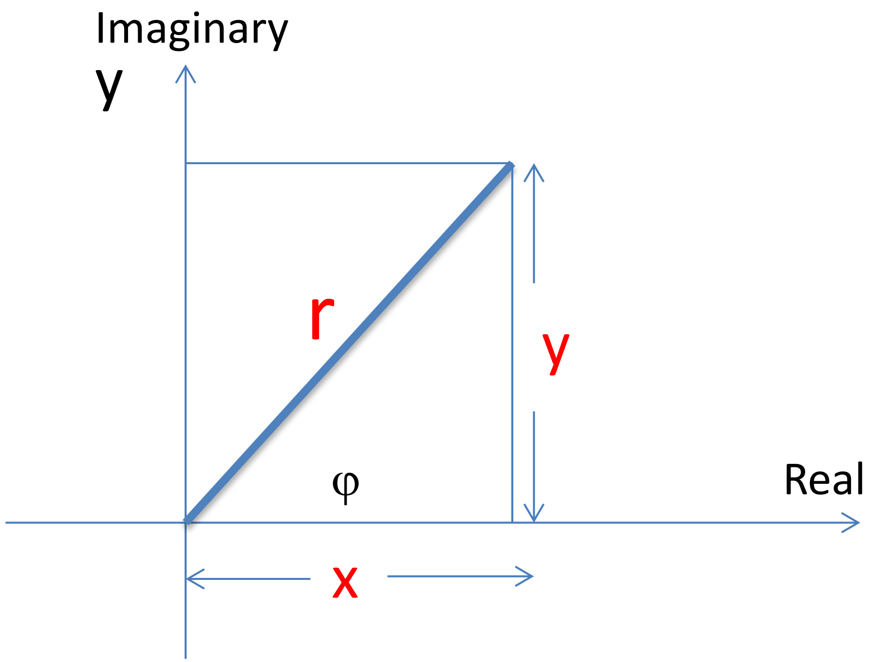
$$z = x + jy$$

$$r = \sqrt{x^2 + y^2}$$

φ = ATAN (y/x)

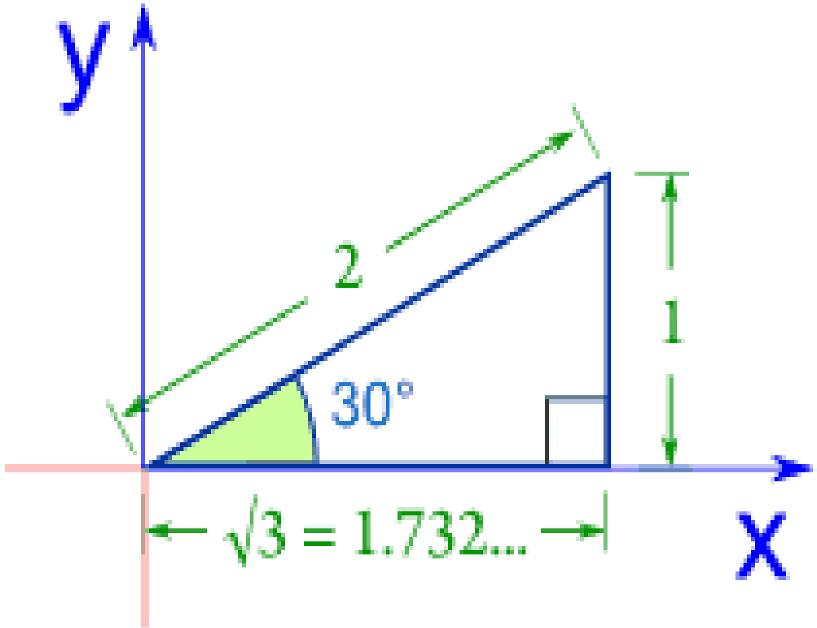
$$\cos(\phi) = \frac{x}{r}$$

$$\sin(\phi) = \frac{y}{r}$$



Must check the sign of sin(φ) and cos(φ) in order to determine the correct quadrant sign of the angle

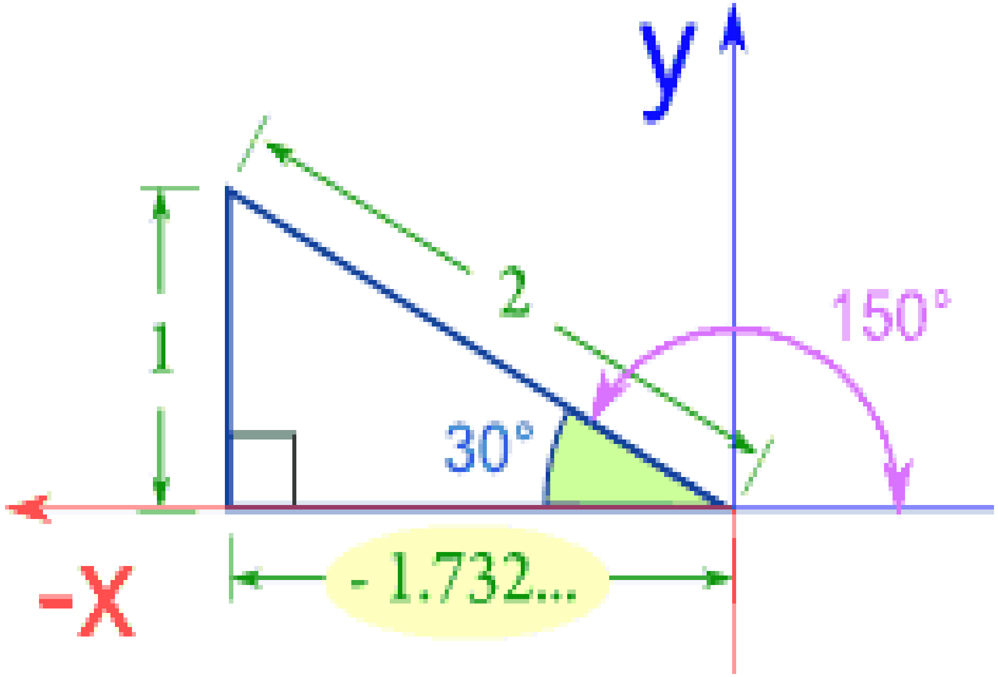
Angle In the first quarter



Sine	sin(30°) = 1 / 2 = 0.5
Cosine	cos(30°) = 1.732 / 2 = 0.866
Tangent	tan(30°) = 1 / 1.732 = 0.577

If both sin and cos are positive, then φ=ATAN(angle)

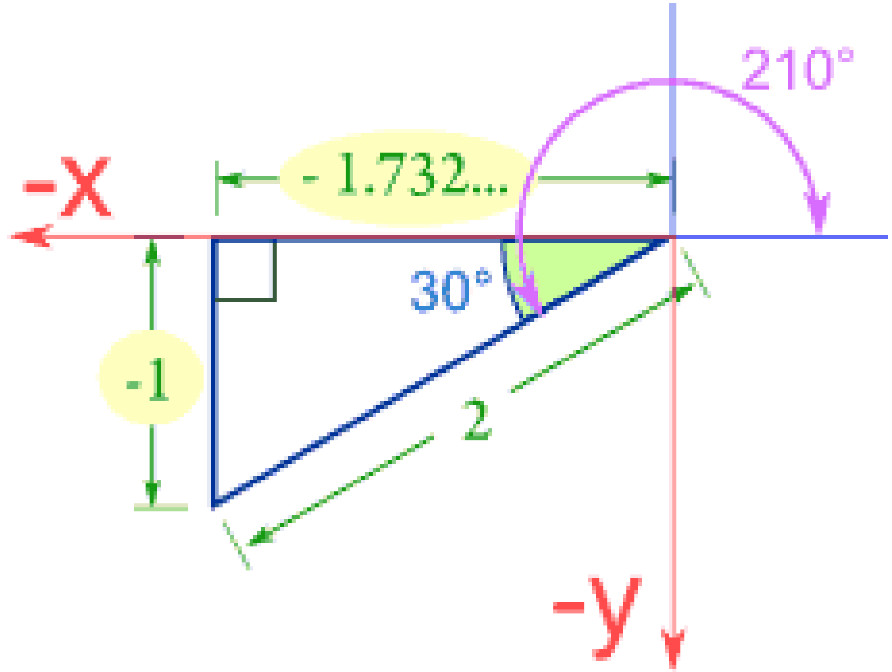
Angle In the second quarter



Sine	sin(150°) = 1 / 2 = 0.5
Cosine	cos(150°) = −1.732 / 2 = −0.866
Tangent	tan(150°) = 1 / −1.732 = −0.577

If sin is +ve and cos is –ve , then
φ=180+TAN(angle)

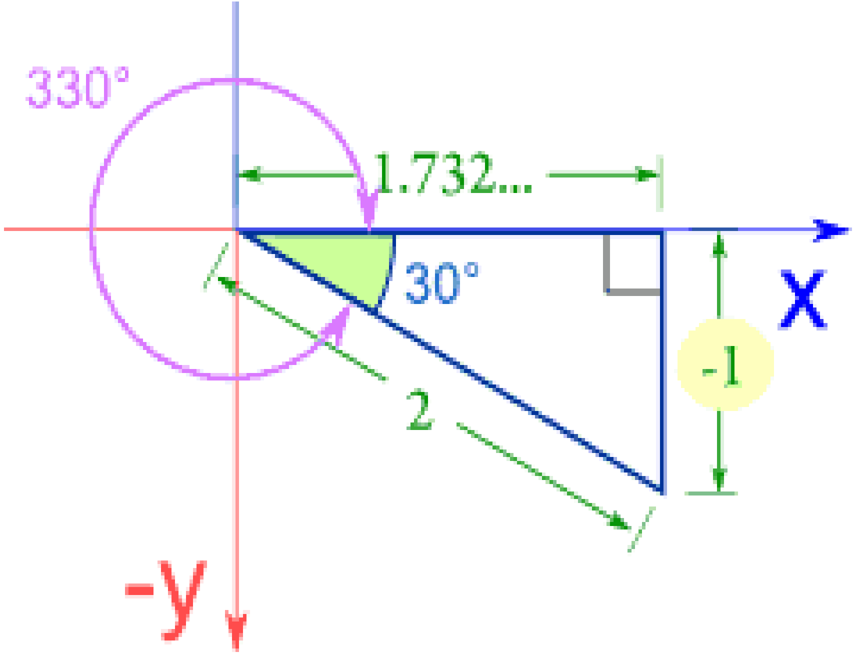
Angle In the third quarter



Sine	sin(210°) = −1 / 2 = −0.5
Cosine	cos(210°) = −1.732 / 2 = −0.866
Tangent	tan(210°) = −1 / −1.732 = 0.577

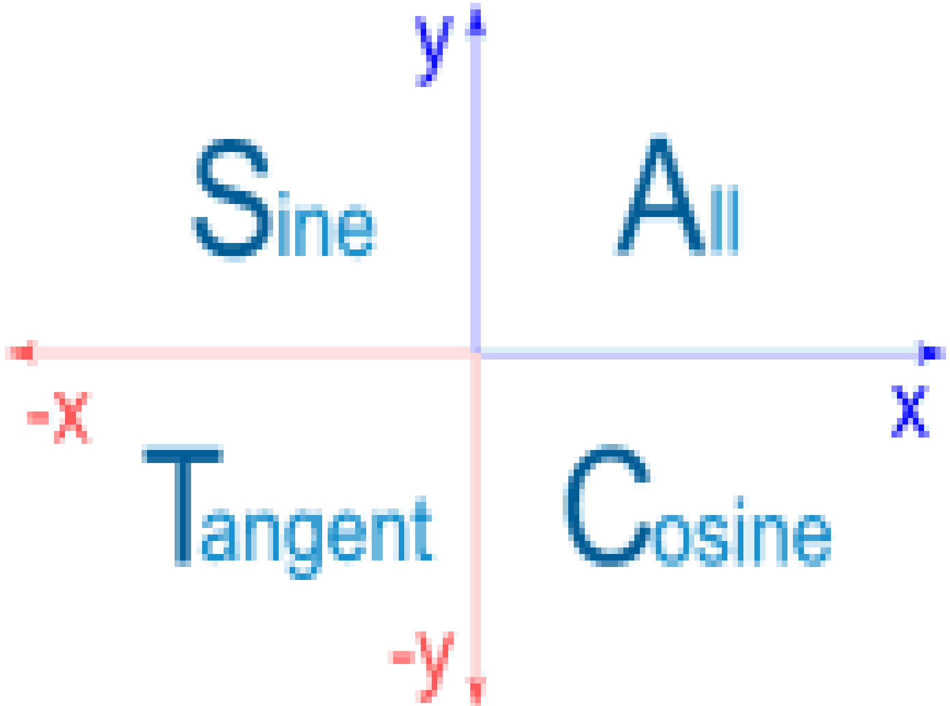
If sin is -ve and cos is –ve , then
φ= -180-TAN(angle)

Angle In the forth quarter



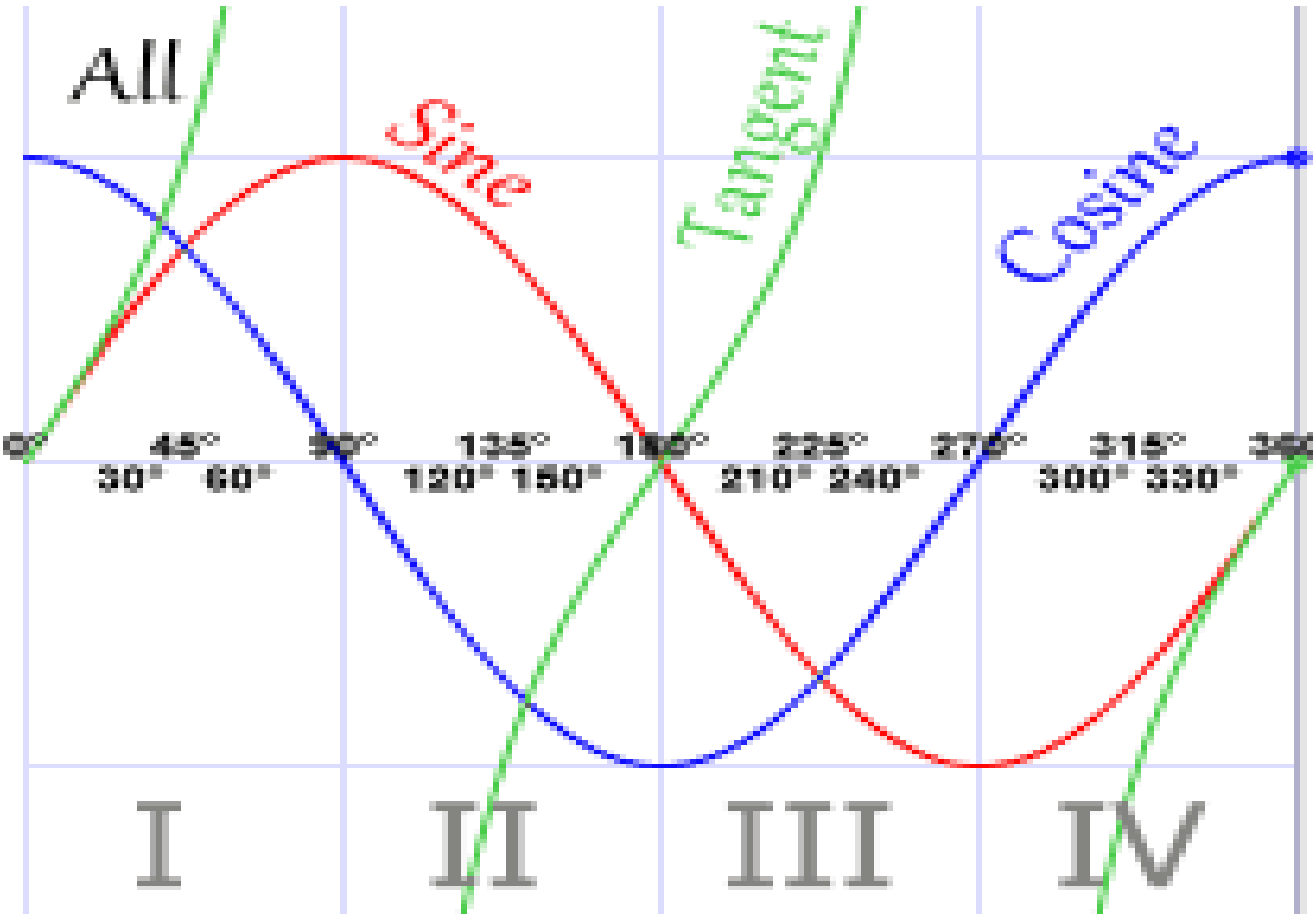
Sine	sin(330°) = −1 / 2 = −0.5
Cosine	cos(330°) = 1.732 / 2 = 0.866
Tangent	tan(330°) = −1 / 1.732 = −0.577

If sin is -ve and cos is +ve , then φ= TAN(angle)



- **All** three of them are positive in **Quadrant I**
- **Sine** only is positive in **Quadrant II**
- **Tangent** only is positive in **Quadrant III**
- **Cosine** only is positive in **Quadrant IV**

Signs of angles in different quadrants



Applications to first and second order systems exposed to periodic excitation input (i.e. Frequency Response)

Frequency response for the first order system

Transfer function for first order system

$$G(s) = \frac{1}{1 + \tau s}$$

For frequency response let s=jω

$$G(j\omega) = \frac{1}{1 + \tau\omega j}$$

Multiply by the conjugate of the complex number

$$G(j\omega) = \frac{1}{1 + \tau\omega j}\frac{1 - \tau\omega j}{1 - \tau\omega j} = \frac{1}{1 + (\tau\omega)^2}(1 - \tau\omega j)$$

$$x = 1 \qquad y = -\tau\omega$$

$$\tan(\phi) = \left(\frac{y}{x}\right) = -\tau\omega$$

$$r = \sqrt{1 + (\tau\omega)^2}$$

$$\sin(\phi) = \frac{y}{r} = \frac{-\tau\omega}{\sqrt{1 + (\tau\omega)^2}}$$

$$cos(\phi) = \frac{x}{r} = \frac{1}{\sqrt{1 + (\tau\omega)^2}}$$

Frequency response for the second order system

Differential equation

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = F(t)$$

Laplace transfer

$$(\frac{m}{k}s^2 + \frac{c}{k}s + 1)Y(s) = F(s)$$

General Transfer function

$$Y(s) = \frac{F(s)/k}{\frac{m}{k}s^2 + \frac{c}{k}s + 1}$$

Definition of natural frequency and damping ratio

$$\omega_n = \sqrt{\frac{k}{m}} \qquad \zeta = \frac{c}{2\sqrt{km}}$$

Transfer function ratio

$$G(s) = \frac{1}{\left[\left(\frac{\omega}{\omega_n}\right)^2 s^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)s + 1\right]}$$

Transfer function for periodic input . Change s to jω

$$G(j\omega) = \frac{1}{\left[\left(\frac{\omega}{\omega_n}\right)^2 (j)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)j + 1\right]}$$

Frequency response for the second order system

$$G(j\omega)=\frac{1}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2+\left(\frac{2\zeta\omega}{\omega_n}\right)j\right]}$$

Since $j^2=-1$

Multiply by the conjugate of the complex number

$$G(j\omega)=\frac{1}{\left[\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]+\left(\frac{2\zeta\omega}{\omega_n}\right)j\right]}*\frac{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]-\left(\frac{2\zeta\omega}{\omega_n}\right)j}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]-\left(\frac{2\zeta\omega}{\omega_n}\right)j}$$

From this equation one can get the amplitude ratio and phase shift

$$G(j\omega)=\frac{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]-\left(\frac{2\zeta\omega}{\omega_n}\right)j}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

$$x=1-\left(\frac{\omega}{\omega_n}\right)^2 \qquad y=-\frac{2\zeta\omega}{\omega_n} \qquad r=\sqrt{x^2+y^2}$$

$$\sin(\phi)=\frac{y}{r} \qquad \cos(\phi)=\frac{x}{r} \qquad \tan(\phi)=\frac{y}{x}$$

Frequency response for the second order system

$$G(j\omega) = \frac{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right] - \left(\frac{2\zeta\omega}{\omega_n}\right) j}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

Amplitude ratio

$$|G(j\omega)| = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

Phase shift

$$\phi = ATAN\left[\frac{-\frac{2\zeta\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$$x = 1 - \left(\frac{\omega}{\omega_n}\right)^2 \qquad y = -\frac{2\zeta\omega}{\omega_n} \qquad r = \sqrt{x^2 + y^2}$$

$$\sin(\phi) = \frac{y}{r} \qquad \cos(\phi) = \frac{x}{r} \qquad \tan(\phi) = \frac{y}{x}$$